

Problem Set 1 Solutions

CS&SS Math Camp 2021

August 10, 2021

1. $\sum_{k=1}^4 (k-1)^2 = (1-1)^2 + (2-1)^2 + (3-1)^2 + (4-1)^2 = 0 + 1 + 4 + 9 = 14$

2. $\prod_{i=2}^{10} \frac{(i+1)}{i} = \frac{3}{2} \frac{4}{3} \frac{5}{4} \cdots \frac{10}{9} \frac{11}{10} = \frac{11}{2}$

3. $\log(e^2) = 2$

4. $e^4 e^{10} = e^{14}$

5. $10^3 10^{-2} = 10^{3-2} = 10$

6. $400^{\frac{1}{2}} = 20$

7. Compute the root(s) of the following quadratic equation:

$$x^2 - 8x + 12 = 0$$

$$(x - 6)(x - 2) = 0$$

$$x = 6 \text{ or } x = 2$$

8. Compute the root(s) of the following quadratic equation:

$$x^2 + 5x + 4 = 0$$

$$(x + 4)(x + 1) = 0$$

$$\Rightarrow x = -4, x = -1$$

OR

Using the quadratic equation,

$$\begin{aligned}x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\x &= \frac{-5 + \sqrt{5^2 - 4 \times 1 \times 4}}{2 \times 1} \\x &= \frac{-5 + \sqrt{25 - 16}}{2} \\x &= \frac{-5 + \sqrt{9}}{2} \\x &= \frac{-5 + 3}{2} \\x &= \frac{-2}{2} \\x &= -1 \\x &= \frac{-5 - \sqrt{5^2 - 4 \times 1 \times 4}}{2 \times 1} \\x &= \frac{-5 - \sqrt{9}}{2} \\x &= \frac{-5 - 3}{2} \\x &= \frac{-8}{2} \\x &= -4\end{aligned}$$

$$x = -4 \text{ or } x = -1$$

9. Suppose the supply curve for oil is expressed with the following linear equation:

$$-x + 4y = 30$$

And the demand curve is expressed with this equation:

$$2x + 5y = 9$$

Solve the system of linear equations to compute the equilibrium cost. Plot the two lines. Multiply the first equation by two:

$$2(-x + 4y = 30)$$

$$-2x + 8y = 60$$

Add the two equations

$$-2x + 8y = 60$$

$$+2x + 5y = 9$$

$$0x + 13y = 69$$

$$y = \frac{69}{13}$$

Substitute back in to solve for x:

$$2x + 5\left(\frac{69}{13}\right) = 9$$

$$2x + \frac{345}{13} = 9$$

$$2x = \frac{-228}{13}$$

$$x = \frac{-114}{13}$$

10. Compute the limit:

$$\lim_{x \rightarrow \infty} x^4 = \infty$$

11. Compute the limit:

$$\lim_{x \rightarrow 2} x^4 = 2^4 = 16$$

12. Compute the limit:

$$\lim_{x \downarrow 3} \frac{1}{x - 3} = \infty$$