

# Problem Set 6 Solutions

## CS&SS Math Camp 2021

1. Let  $Y$  be a uniform random variable on the interval  $[2,10]$ .

(a) Compute the expected value ( $E[Y]$ ).

$$E[X] = \int_2^{10} x \cdot \frac{1}{10-2} dx = \frac{1}{8} \int_2^{10} x dx = \frac{1}{8} [x^2/2]_2^{10} = \frac{1}{8} [100/2 - 4/2] = 6$$

(b) Compute the variance ( $Var[Y]$ ).

$$V[X] = \frac{1}{8} \int_2^{10} (x-6)^2 dx = \frac{1}{8} \cdot \frac{1}{3} [(x-6)^3]_2^{10} = \frac{1}{24} [(10-6)^3 - (2-6)^3] = 5.33\bar{3}$$

2. A family has 4 pets, let  $X$  denote the number of cats. Assume the only pets are cats or dogs and they are assigned to families in equal probability.

(a) Write down the probability distribution of  $X$ . Hint: start by writing down the sample space and count the number of ways each event could occur. Slide 10 from Lecture 6 should be helpful.

$X$ -Value	0	1	2	3	4
$P(X = x_i)$	1/16	4/16	6/16	4/16	1/16

(b) Compute the expected value ( $E[X]$ ).

$$E[X] = 0(1/16) + 1(4/16) + 2(6/16) + 3(4/16) + 4(1/16) = 2$$

(c) Compute the variance ( $Var[X]$ ).

$$V[X] = (0-2)^2(1/16) + (1-2)^2(4/16) + (2-2)^2(6/16) + (3-2)^2(4/16) + (4-2)^2(1/16) = 1$$

3. Toss a coin 4 times, let  $X$  denote the number of heads.

(a) Write down the probability distribution of  $X$ . Hint: start by writing down the sample space and count the number of ways each event could occur. Slide 10 from Lecture 6 should be helpful.

$X$ -Value	0	1	2	3	4
$P(X = x_i)$	1/16	4/16	6/16	4/16	1/16

(b) Compute the expected value ( $E[X]$ ).

$$E[X] = 0(1/16) + 1(4/16) + 2(6/16) + 3(4/16) + 4(1/16) = 2$$

(c) Compute the variance ( $Var[X]$ ).

$$V[X] = (0-2)^2(1/16) + (1-2)^2(4/16) + (2-2)^2(6/16) + (3-2)^2(4/16) + (4-2)^2(1/16) = 1$$

4. John pays \$40 per year for towing insurance. He thinks the probability that he will need to have his car towed is 10% and the probability that he will need to have it towed more than once is zero. Without insurance the cost of towing is \$100, but the cost is zero if insured. Let  $X$ =John's expenses next year for towing and/or insurance.

(a) If he buys insurance, what is the value of  $X$ ?

$$X = \$40$$

(b) If he doesn't buy insurance, what two values can  $X$  take?

$$X = \$0 \text{ or } \$100$$

(c) Find  $E[X]$  for both (a) and (b). Should he buy the insurance?

Buys insurance:  $E[X] = 1 \cdot 40 = \$40$

Does not buy insurance:  $E[X] = 0.1 \cdot 100 + 0.9 \cdot 0 = \$10$

He expects to pay less if he doesn't buy insurance, so he should not buy it.

5. Let  $X$  represent the number of jobs held during the past year for students at a school, and suppose  $X$  has the following probability distribution:

$X$ -Value	0	1	2	3	4
$P(X = x_i)$	0.15	0.28	0.36	0.10	0.11

(a) What is the probability that a randomly selected student has fewer than two jobs?

$$P(X = 0) + P(X = 1) = 0.15 + 0.28 = 0.43$$

(b) Find  $P(X > 0)$ .

$$P(X > 0) = 1 - P(X = 0) = 1 - 0.15 = 0.85$$

(c) Find  $P(X > 2)$ .

$$P(X > 2) = P(X = 3) + P(X = 4) = 0.10 + 0.11 = 0.21$$