Problem Set 7: Discrete Distributions Solutions
CS&SS Math Camp 2021

1. What is the proper distribution for the following random variables? What parameters do you need for the distribution?

   (a) Draw 4 cards from a deck, X =the number of hearts.

   X ~HyperGeometric, where N=52 (number of cards), n=4 (number of draws), and K=13 (number of hearts).

   (b) Observe the weather in Seattle for 7 days. Y =number of sunny days.

   X ~Binomial(n,p), where n=7, and p=probability of sunny. (It is possible to approximate this process with a Poisson distribution, however this approximation is generally only appropriate when p is small and n is large.)

   (c) Take the bus to school each day for 30 days. X = number of times the bus is late.

   X ~Binomial(n,p), where n=30, and p=probability of late bus. (It is possible to approximate this process with a Poisson distribution, however this approximation is generally only appropriate when p is small and n is large.)

   (d) Survey 100 people and ask which candidate they will vote for, among 4 candidates. X = the number of votes for each candidate.

   X ~Multinomial, where n=100 and $p_1 - p_4$ is the probability of voting for each of the 4 candidates.

2. Let $X \sim Bin(n = 3, p = 0.5)$.

   (a) Write down the probability mass function for X.

$$P(X = x|n = 3, p = 0.5) = \binom{3}{x} 0.5^x (1 - 0.5)^{3-x} = \binom{3}{x} 0.5^3$$

(b) Graph the distribution of X.

Figure 1 displays the probability distribution (or mass function) for a Binomial(3,0.5).

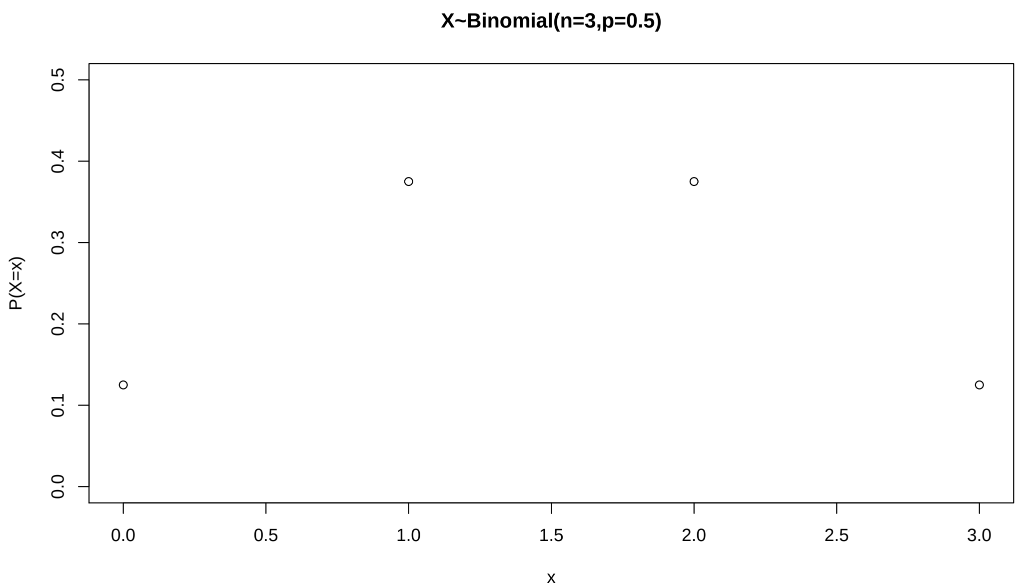

Figure 1: Probability Distribution of a Binomial(3,0.5)

(c) $E[X]$

$$E[X] = np = 3 \cdot 0.5 = 1.5$$

(d) $V[X]$

$$V[X] = np(1-p) = 3 \cdot 0.5 \cdot 0.5 = 0.75$$

3. Suppose the probability that you pass your graduate school qualifying exam is 75%. Let X be the number of tries until you pass.

   (a) What distribution would you use to model X?

$X \sim$ Geometric(p=0.75). Remember, you can think of the Geometric distribution two different ways. (1) X=the number of the trial with the first success (see lecture 7, slide 16). (2) X=the number of failures before a success (see lecture 7, slide 18). The distribution depends on the way you parameterize X.

(b) $P(X = 1) =$

(1) $0.75 \cdot 0.25^{1-1} = 0.75$

(2) $0.75 \cdot 0.25^{1} = 0.188$

(c) $P(X = 2) =$

(1) $0.75 \cdot 0.25^{2-1} = 0.188$

(2) $0.75 \cdot 0.25^{2} = 0.047$

(d) $P(X > 2) =$

(1) $P(X \leq 2) = 1 - [P(X = 1) + P(X = 2)] = 1 - [0.75 + 0.188] = 0.062$

(2) $P(X \leq 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)] = 1 - [0.75 + 0.188 + 0.047] = 0.015$

4. A Poisson distribution is used to model traffic accidents at an intersection. $X =$ the number of accidents in a month. Assume $X \sim Poisson(\lambda = 1)$.

(a) $P(X = 1) =$

$$\frac{e^{-1} \cdot 1^1}{1!} = 0.368$$

(b) $P(X = 0) =$

$$\frac{e^{-1} \cdot 1^0}{0!} = 0.368$$

(c) $P(X > 0) =$

$$1 - P(X = 0) = 1 - 0.368 = 0.632$$

(d) Write out the summation (using Σ) that would be used to calculate $E[X]$. (You do not need to solve the summation.)

$$E[X] = \sum_{k=0}^{\infty} k \cdot P(X = k) = \sum_{k=0}^{\infty} k \cdot \frac{e^{-1} 1^k}{k!} = \sum_{k=0}^{\infty} \frac{1}{(k-1)! \cdot e}$$